



聴覚系の時間-周波数分析におけるwavelet変換からの逸脱について

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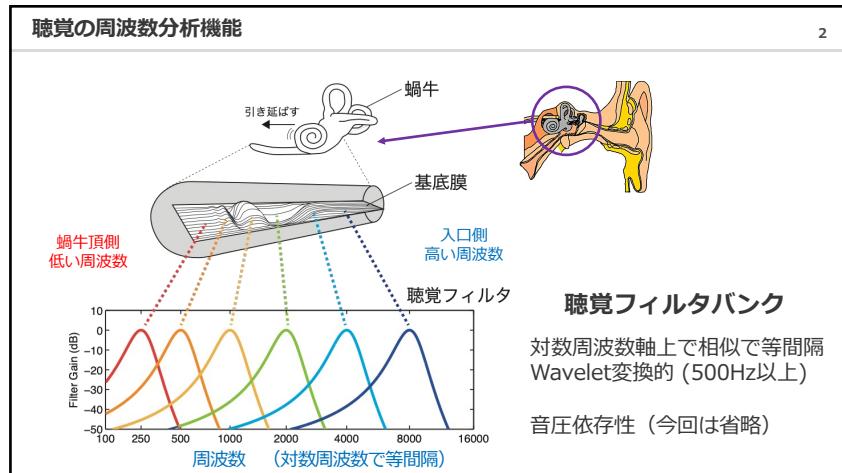
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聴覚末梢系の解剖学と機能

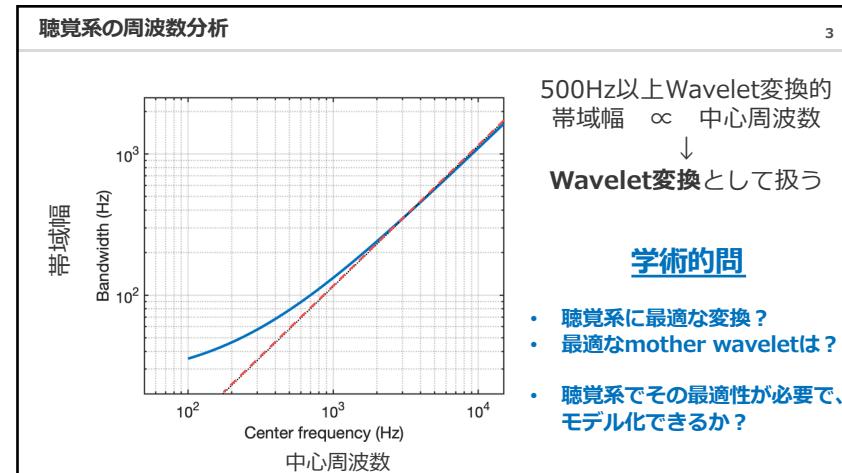
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1) 振動→神経発火
2) 周波数分析
○ Wavelet変換
× Fourier変換

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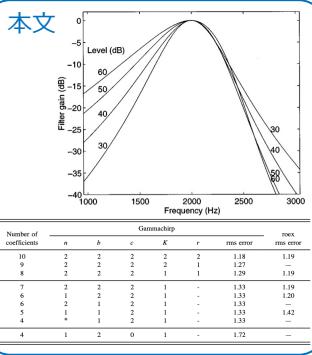
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初出の論文 1997年

Irino, T and Patterson, R.D.
J. Acoust. Soc. Am., 101 (1), pp.412-419, January 1997. [doi:10.1121/1.417975] 4

A time-domain, level-dependent auditory filter: The gammachirp

本文



付録

$$\frac{d}{dt} A(t) + \frac{1}{2(\Delta t)} I(t) \left(-j(\omega) - \frac{(t)}{2(\Delta t)} \right) A(t) = 0, \quad (\text{A9})$$

The nominal solution is

$$A(t) = \exp \left[-\frac{1}{4(\Delta t)^2} t^2 + \frac{(t)}{2(\Delta t)^2} + j(\omega)t \right], \quad (\text{A10})$$

= $a' \exp \left[-\frac{1}{4(\Delta t)^2} (t-t_0)^2 + j(\omega)t \right]$ where $a' = \sqrt{\lambda}/2(\Delta t)^2$, $t_0 = -c_1/\omega$, $c_1 = \ln(a)/2(\Delta t)^2$ andwhere $\lambda = \langle \langle \mathcal{T}, \mathcal{W} \rangle \rangle 2(\Delta t)^2 = j(\omega)/2(\Delta t)^2$.

Equation (A9) expands to

$$\left[-j \frac{d}{dt} \right] A(t) - (a_1 + ja_2) A(t) + (-c_1 + ja_2) t A(t) = 0, \quad (\text{A11})$$

where $a_1 = j(\omega)/2(\Delta t)^2$, $a_2 = -c_1/\omega$, $c_1 = \ln(a)/2(\Delta t)^2$ and $c_1 = \operatorname{Re}(c_1) - c_1 \operatorname{Im}$. The solution is $A(t) = \exp \left[-a' t + j(\omega_0 t + \phi) \right], \quad (\text{A12})$ where a' is a constant. The envelope $\exp(t^2 \exp(-a'/2))$ is agamma distribution function $\gamma(t)$. The instantaneous frequency is $\omega_0 + 2\pi a'/t$. When played as a sound, the carrier would be a chirp, and hence the name "chirp". The envelope $\exp(-a'/2)$ in equation (A12) becomes a gamma function. Thus, the gamma function is a first order approximation to the gammachirp function.

G. The Mellin operator

Mellin operator introduced the concept of a scale operator into signal processing in the form

$$\mathcal{C} = [\mathcal{T} \mathcal{W} + \mathcal{W} \mathcal{T}] / 2. \quad (\text{A13})$$

Previously it had been known as the operator representing an affine variable in certain mechanics (Klauder, 1980). The

論文被引用数 (2025/2末)
392件 (Google scholar)
177件 (Web of science)

まあ、そこそこ

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最小不確定性： 時間-周波数空間

Cohen, Time-Frequency Analysis (1994)

時間-周波数空間の不確定性

Time operator, \mathcal{T} , Frequency operator, \mathcal{W}

$$\Delta t \cdot \Delta \omega \geq \frac{1}{2} |\langle \mathcal{T}, \mathcal{W} \rangle| = \frac{1}{2}$$

最小不確定性関数 : Gabor/Gauss

$$g_b(t) = a \exp \left\{ -\frac{(t - \langle t \rangle)^2}{4(\Delta t)^2} \right\} \exp(j\langle \omega \rangle t)$$

$$= a \exp \left\{ -\frac{(t - \mu)^2}{2\sigma^2} \right\} \exp(j\omega_0 t)$$

→ これは、聴覚フィルタ特性と異なる

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最小不確定性： 時間-スケール空間 Cohen, Time-Frequency Analysis (1994)

湯川英樹, joint operator 確かに言及してたが深せてない

Mellin(Scale)変換

Waveletでスケール変形しても、同じ分布と表現される

Mellin operator: $\mathcal{C} := (\mathcal{T} \cdot \mathcal{W} + \mathcal{W} \cdot \mathcal{T})/2$

$$\text{最小不確定性: } \Delta t \cdot \Delta c \geq \frac{1}{2} |\langle \mathcal{T}, \mathcal{C} \rangle| = \frac{1}{2} |\langle t \rangle|$$

最小不確定性関数： Gammachirp

$$g_c(t) = a t^{\alpha_2 + jc_1} \exp(-\alpha_1 t + j\omega_0 t)$$

聴覚末梢系の知見を入れて…

$$g_c(t) = a t^{n-1} \exp(-b \operatorname{ERB}_N(f_r) t) \cos(2\pi f_r t + c \ln t + \phi)$$

→ 聴覚フィルタ実験データに適合良し

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知覚系におけるスケール変換： 存在？ 有効？

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スケール不変性 (式で書くよりざっくり…)

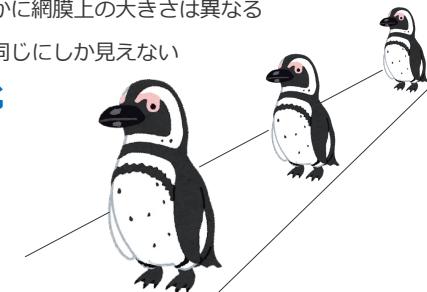
- 視覚系：寸法が異なっても、同じに見える

• 距離が異なれば明らかに網膜上の大きさは異なる

•しかし、同じものは同じにしか見えない

→ 寸法の正規化

- 聴覚系・音では？



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普段の経験： 音声からの情報抽出

8

顔が見えない電話音声を聞いても…



聴覚系における寸法知覚の計算理論の構築

8

生態学的な背景：寸法知覚は人間だけの得意技ではない

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[Charlton+, 2012] [Terranova+, 2023]



体が大きい=強い♂

寸法： 大（捕食者）vs 小（逃避者）

- 哺乳類：夜行性で音から判断必要
- 遠くからでも、木陰でもわかる必要
- 視覚による寸法推定は、周囲との対比が必要であってならない

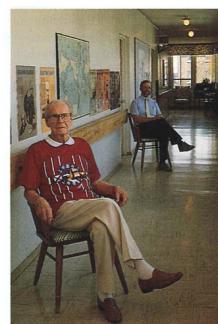
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視覚系における寸法知覚の錯覚

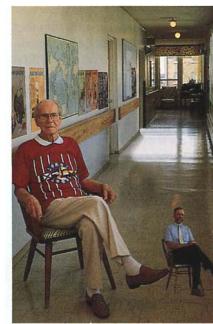
Kohn, 2014, The science of memory (ppt)
[Source photo: Ed van der Elsken?]

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後の男性は、同じくらいの身長



同じ画像も位置によって、小人に？



→ 音声を聞いた方が信用できる

視覚の錯覚：多数…だましやすい
聴覚の錯覚：作成難しく少数

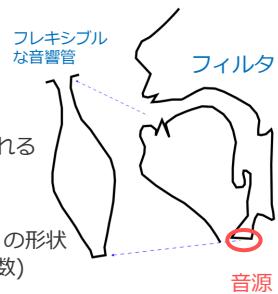
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音声生成の基礎

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ソースフィルタ理論 [Fant, 1970]

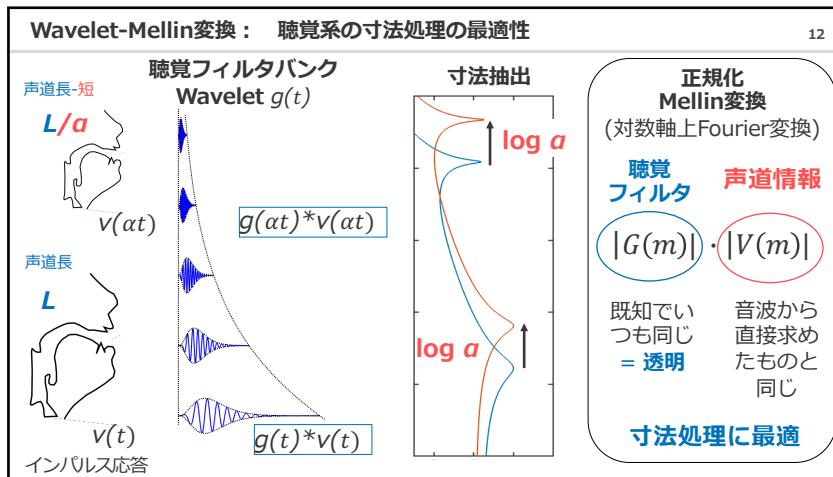
- 母音の生成
 - ソース(音源)**：声帯振動のパルス列
 - 声の高さの情報
 - 基本周波数 F_0 の倍音成分がたくさん含まれる
 - 声道フィルタ**：
 - 音韻性の情報
 - フレキシブルな音響管(声道, Vocal tract)の形状に依存した共鳴周波数(=ホルマント周波数)
 - ホルマント周波数 $F_1 \cdot F_2 \cdot F_3 \dots$



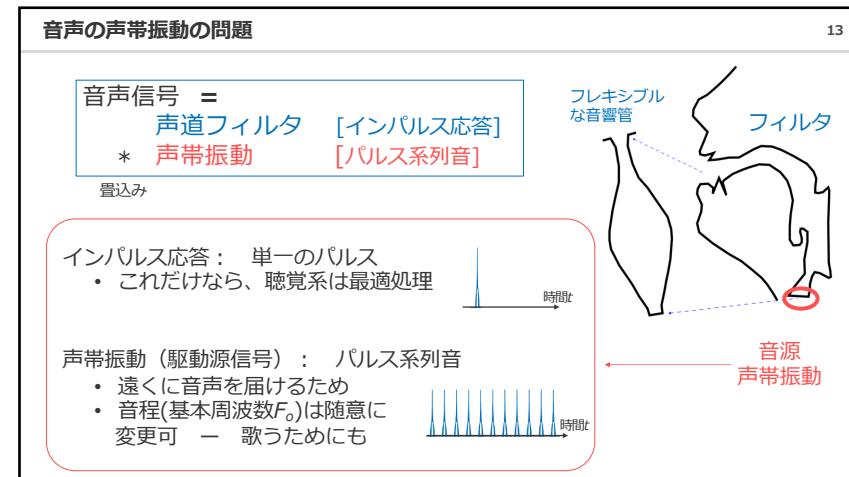
$$\text{音声信号} = \text{声道フィルタ} (\text{声道形状} \cdot \text{声道長}) * \text{声帯振動} \\ [\text{インパルス応答}] * [\text{パルス系列音}]$$

畳込み

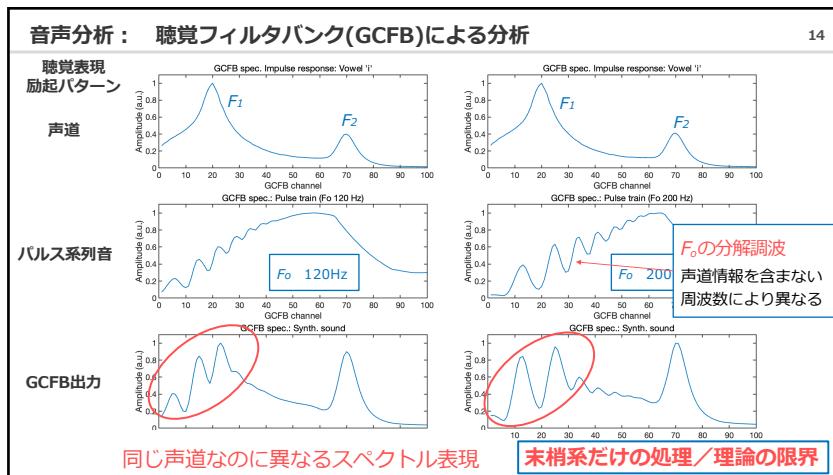
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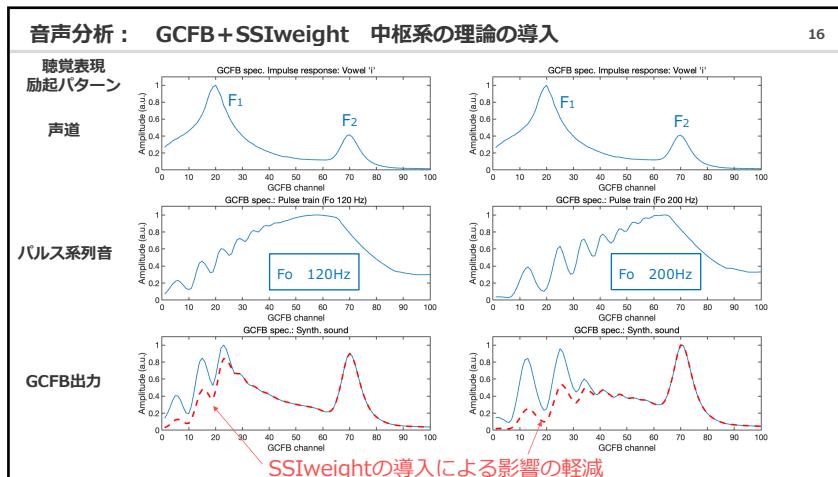
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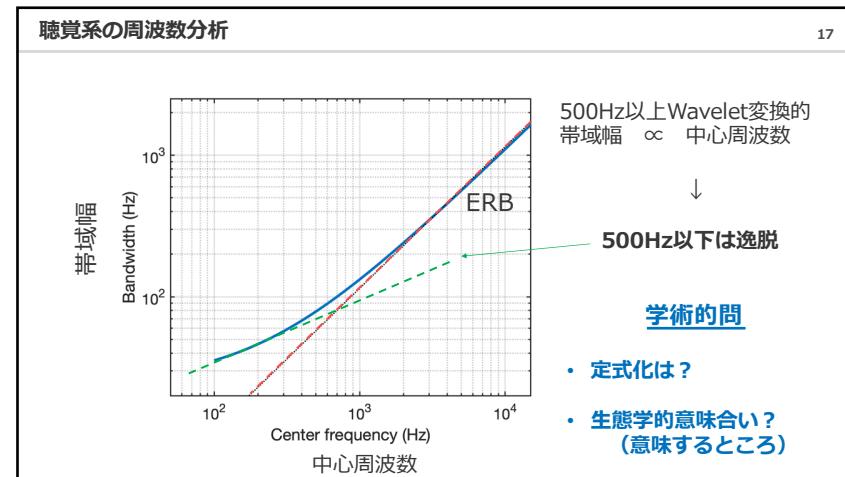
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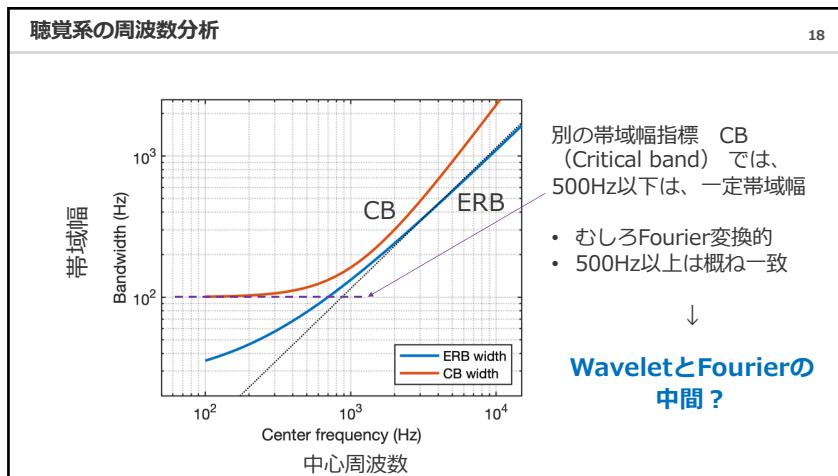
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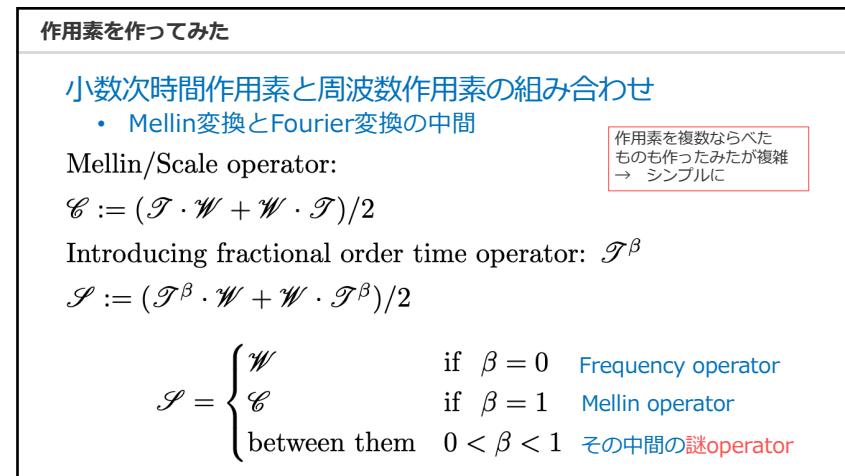
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AI GPT-5 (Copilot) に入れてみた

不確定性に関して

$$\Delta A \cdot \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|$$

$$A = \mathcal{T}, \quad B = \mathcal{S} := \frac{1}{2}(\mathcal{T}^\beta \mathcal{W} + \mathcal{W} \mathcal{T}^\beta)$$

$[\mathcal{T}, \mathcal{W}] = i, \quad [\mathcal{T}, \mathcal{T}^\beta] = 0$

$$\begin{aligned} [\mathcal{T}, \mathcal{S}] &= \frac{1}{2}([\mathcal{T}, \mathcal{T}^\beta \mathcal{W}] + [\mathcal{T}, \mathcal{W} \mathcal{T}^\beta]) \\ &= \frac{1}{2}([\mathcal{T}, \mathcal{T}^\beta] \mathcal{W} + \mathcal{T}^\beta [\mathcal{T}, \mathcal{W}] + [\mathcal{T}, \mathcal{W}] \mathcal{T}^\beta + \mathcal{W} [\mathcal{T}, \mathcal{T}^\beta]) \\ &= \frac{1}{2}(\mathcal{T}^\beta(i) + (i)\mathcal{T}^\beta) = \frac{i}{2}(2\mathcal{T}^\beta) = i\mathcal{T}^\beta. \end{aligned}$$

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最小不確定性を満たす関数

$$\Delta \mathcal{T} \cdot \Delta \mathcal{S} \geq \frac{1}{2} |\langle [\mathcal{T}, \mathcal{S}] \rangle| = \frac{1}{2} |\langle i\mathcal{T}^\beta \rangle| = \frac{1}{2} |\langle \mathcal{T}^\beta \rangle|.$$

最小不確定性状態

$$(\mathcal{T} - \langle \mathcal{T} \rangle) \psi = i\lambda (\mathcal{S} - \langle \mathcal{S} \rangle) \psi,$$

$$\lambda t^\beta \frac{d\psi}{dt} + (t - t_0) \psi(t) = 0. \quad \text{書き下すとこの微分方程式}$$

解 $g_s(t) = C \exp \left(-\frac{1}{\lambda} \left(\frac{t^{2-\beta}}{2-\beta} - t_0 \frac{t^{1-\beta}}{1-\beta} \right) \right) \quad \beta \neq 1, 2$

Mellin operator: $\mathcal{C} := (\mathcal{T} \cdot \mathcal{W} + \mathcal{W} \cdot \mathcal{T})/2$

$$\Delta t \cdot \Delta c \geq \frac{1}{2} |\langle \mathcal{T}, \mathcal{C} \rangle| = \frac{1}{2} |\langle t \rangle|$$

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最小不確定性を満たす関数 特殊例

解

- $\beta = 1$ の場合 (時間 \mathcal{T} と scale \mathcal{C}) :

$$g_s(t) = C \exp \left(-\frac{1}{\lambda} \left(\frac{t^1}{1} - t_0 \ln t \right) \right) = Ct^{\frac{t_0}{\lambda}} e^{-\frac{t}{\lambda}}.$$

ガンマチャーブ関数と同じ形式

- $\beta = 0$ の場合 (時間 \mathcal{T} と周波数 \mathcal{W}) :

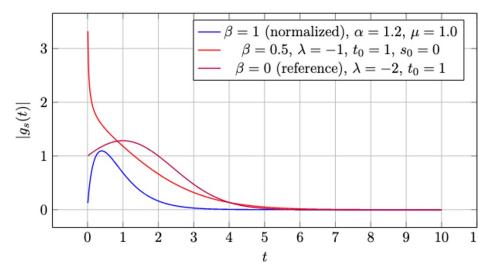
$$g_s(t) = C \exp \left(-\frac{1}{\lambda} \left(\frac{t^2}{2} - t_0 t \right) \right),$$

ガウス関数・ガボール関数に近い。 λ による

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グラフ出力

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Figure 1: External-data plots of $|g_s(t)|$ for different β and parameters.

$\beta=0$: ガウス関数的
 $\beta=1$: ガンマ関数的

$\beta=0.5$: $t=0$ で、発散？

↓
帯域幅が広がって、使え
なさそう

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文献のsuggestion

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1. Fractional Fourier Transform (FrFT) と音響解析
 - Fractional Fourier Sound Synthesis
 - Relations between Fractional Operations and Time-Frequency Distributions
2. 新しい小数次演算子の導入
 - New Fractional Operators and Their Properties
3. Mellin変換と小数次拡張
 - Fractional Mellin Transform
 - Foundations of Fractional Mellin Transform Analysis

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ついでに論文まで書いてくれた…

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Uncertainty Relation and Minimal States for Fractional Order Time Operator

Toshio Iriko

November 20, 2025

Abstract

We derive the uncertainty relation for the time operator \mathcal{T} and the symmetric product operator $\mathcal{T}^\beta = (\mathcal{D}^\beta \mathcal{T} + \mathcal{T} \mathcal{D}^\beta)/2$. We also derive the corresponding minimal uncertainty states. We give an explicit time-domain differential equation for the minimal state and a closed-form expression $g_\alpha(t)$, including normalization for the case $\beta = 1$, as well as a Mellin-transform representation. A figure illustrates parameter effects.

1 General Uncertainty Relation and Minimal-State Condition

For self-adjoint operators A, B and a normalized state ψ in a common dense domain, the uncertainty relation reads

$$\Delta A \cdot \Delta B \geq \frac{1}{2} |([A, B])|, \quad (1)$$

with $\Delta A := \sqrt{(\langle A - \langle A \rangle \rangle)^2}$, $\Delta B := \sqrt{(\langle B - \langle B \rangle \rangle)^2}$. Equality holds if and only if the *minimal-state condition*

$$(A - \langle A \rangle)\psi = i\lambda(B - \langle B \rangle)\psi, \quad \lambda \in \mathbb{R}, \quad (2)$$

is satisfied.

2 Setting and Basic Commutators

8 Figure: Shape of $|g_\alpha(t)|$ (External Data)

Below we illustrate $|g_\alpha(t)|$ using PGFPlots with external CSV files produced from the minimal-state formulas:

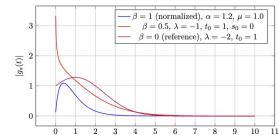


Figure 1: External-data plots of $|g_\alpha(t)|$ for different β and parameters.

9 Remarks on Domains and Self-Adjointness

- The expectation $\langle [A, B] \rangle$ requires the state ψ to be in the common domain of A, B, AB, BA .
- The fractional power \mathcal{T}^β is defined via the spectral theorem assuming $\mathcal{T} \geq 0$; powers of the same operator commute.
- In Mellin-type realizations (scale-domain), analogous results follow by replacing t with t^{β} and using the scale generator $-i\frac{d}{dt^{\beta}\psi}$.
- For $\beta < 2$, choosing $\lambda < 0$ ensures decay of the real exponential factor in (13) as $t \rightarrow \infty$.

まとめと課題

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- 単なる感想: GPT-5 (Copilot)でけっこ出せるものだ…
- 奈良Wavelet研究集会に申し込んでから始めて、2~3日で出せた
- 自分より正確そうだが、不安もあり → 検証求む
- おまけ：結構複雑な式の連続でも、
Latexのコードを出して楽
- 30年前の方が冴えていた?
AIの進展、恐るべし…
使わない手はない
- 新たな学術的問
 - この小数次時間作用素とは何にあたるのか？
 - たとえば、時間変化点・イベントをより正確に求められる？
 - Wavelet変換よりも、何か特別に便利？
 - 聴覚系において、どのような役割でどのように実装？
 - 別の定式化は？

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Fin

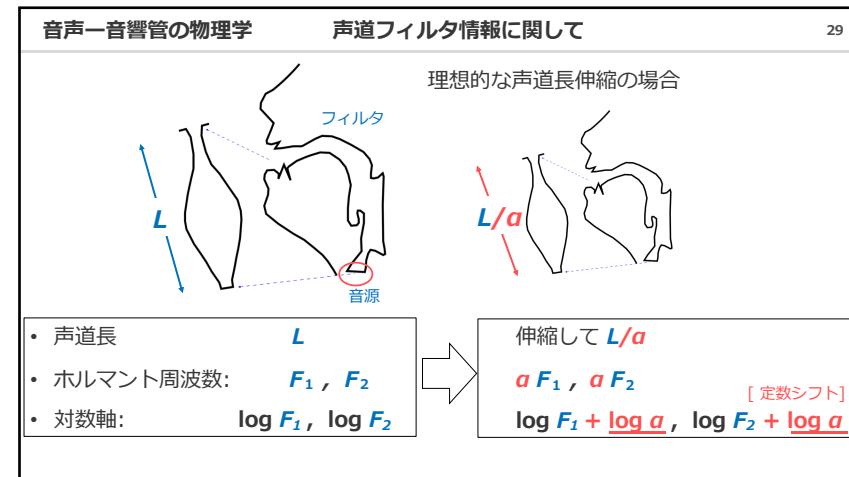
最終講義資料



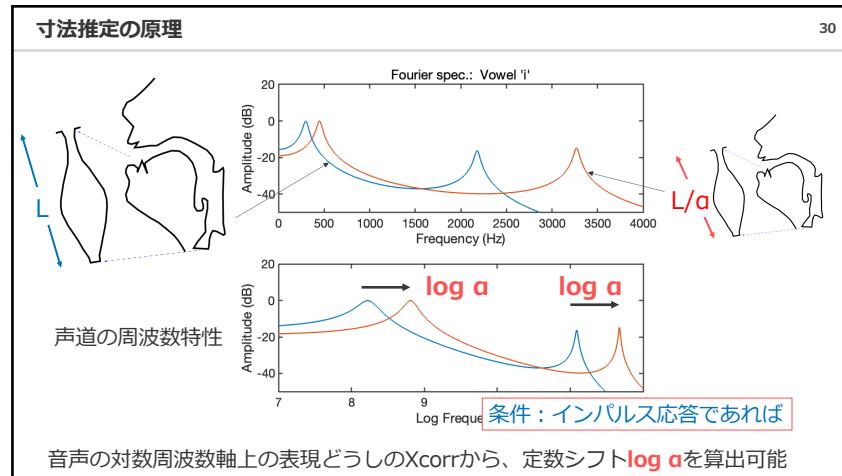
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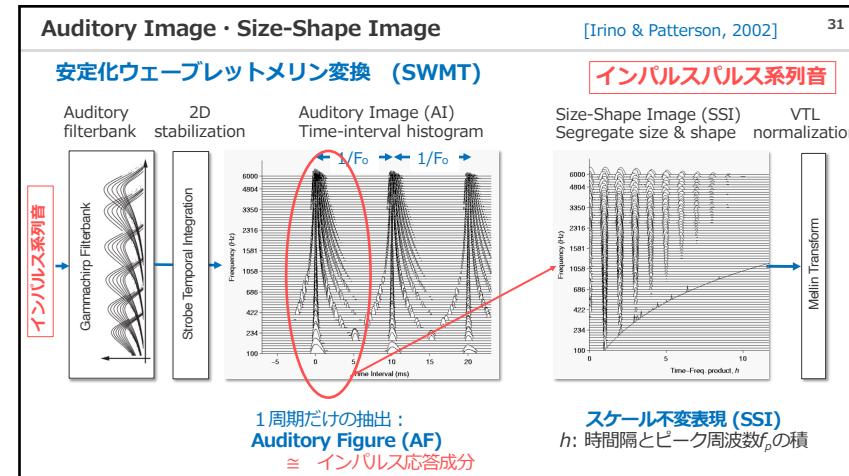
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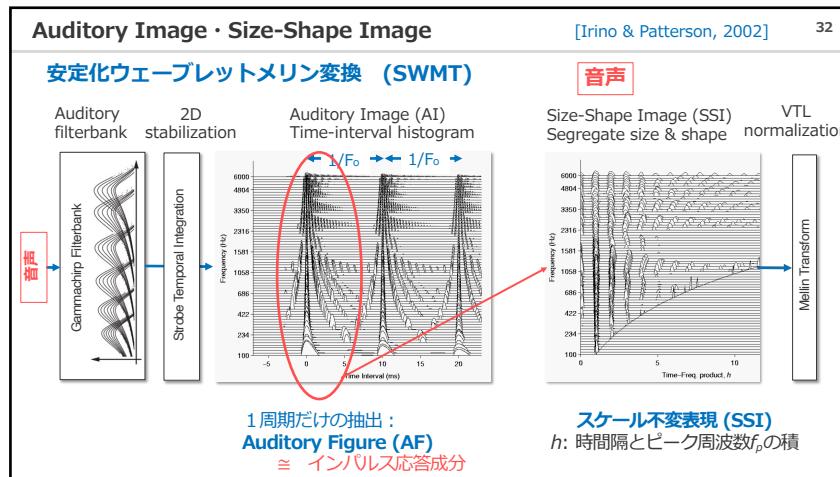
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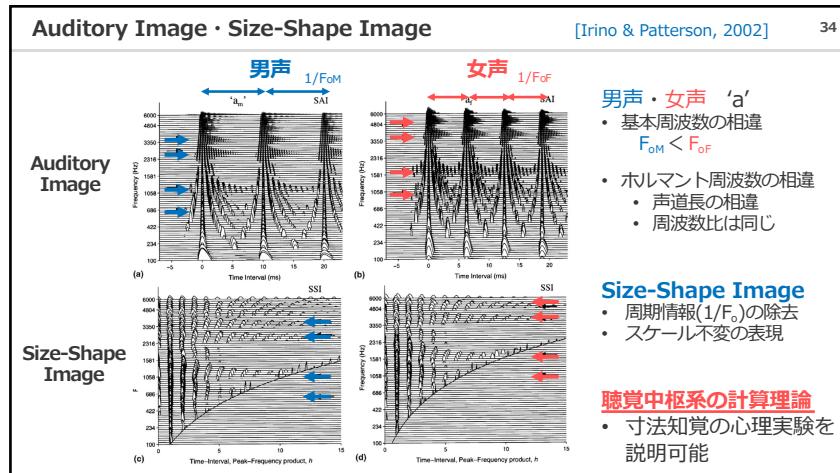
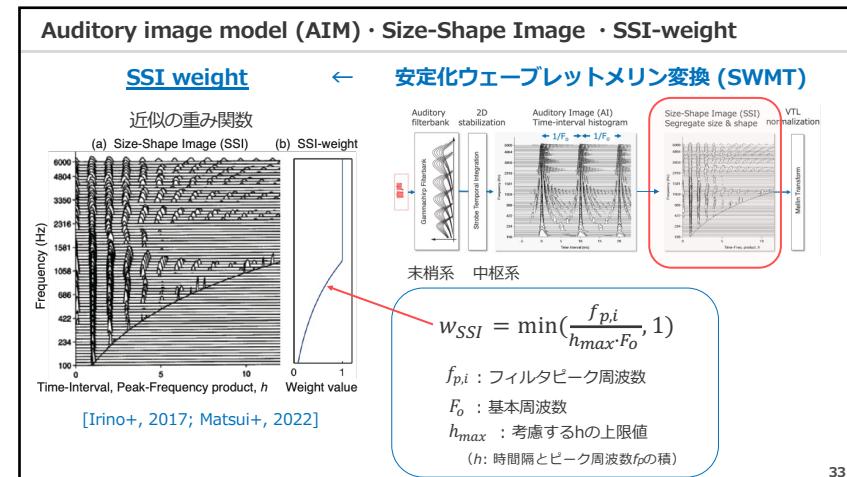
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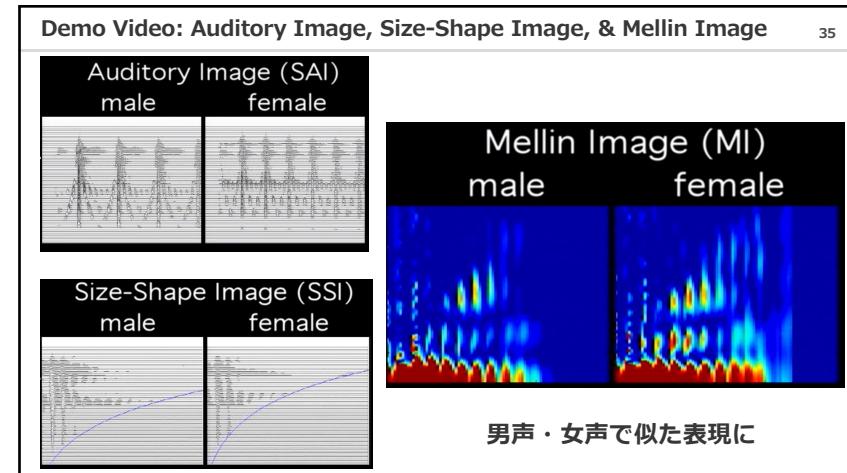
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